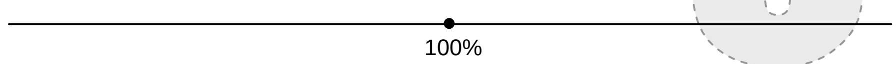
4 PERCENTAGE

The terms ***percent*** means "for every hundred". A fraction whose denominator is 100 is called ***percentage*** and the numerator of the fraction is called the ***rate percent***. Thus, when we say a man made a profit of 20 percent we mean to say that he gained Rs.20 for every hundred rupees he invested in the business, i.e. 20/100 rupees for each Rupee.

The abbreviation of percent is p.c. and it is generally denoted by %.

GENERAL CONCEPTS IN PERCENTAGES:

While dealing with % increase or decrease picturise the following scale in your mind **with reference as 100% in the center.**



An increase by x% implies the value lies on the right hand side of 100% & vice versa.

Let's start with a number X

1. X increased by 10% would become X + 0.1 X = 1.1X
2. X increased by 1% would become X + 0.01 X = 1.01X
3. X increased by 0.1 % would become X + 0.001 X = 1.001X
4. X decreased by 10% would become X – 0.1X = 0.9X
5. X decreased by 1% would become X – 0.01 X = 0.99X
6. X decreased by 0.1% would become X – 0.001 X = 0.999X
7. X increased by 200% would become X + 2X = 3X
8. X decreased by 300% would become X – 3X = – 2X

Example

84% of a particular total is 630 marks. What is 90% equal to?

(1) 750 (2) 675 (3) 450 (4) None of these

Sol. $\frac{84}{100} \times x = 630$ where x is particular total

$x = \frac{63000}{84} = 750$

$\therefore \frac{90}{100} \times 750 = 675.$ **Answer: (2)**

Example

Two numbers are greater than the third number by 25% and 20% respectively. What percent of first number is the second number?

(1) 92% ***(2) 94 %*** ***(3) 96 %*** ***(4) 98 %***

Sol. Let the third number is 100.

Then two numbers are 125, 120.

The first number is $\frac{120}{125} \times 100\%$ i.e. 96% of the second number. **Answer: (3)**

Formulae

1. To express a% as a fraction, divide it by 100 $\Rightarrow$ a% = a/100
2. To express a fraction as %, multiply it by 100 $\Rightarrow$ a/b = [(a/b) $\times$ 100]%
3. Increase % = [Increase / Original value] $\times$ 100%

   Decrease % = [Decrease / Original value] $\times$ 100%

   Change % = [Change / Original value] $\times$ 100%

Toolkit

> If any number (quantity) is changed (increased/decreased) by p%, then
>
> New quantity = Original quantity $\times \left(\frac{100 + p^*}{100}\right)$
>
> * p is (–) ve, when the original quantity is reduced by p%.
> New value = original value + increase
> Or New value = original value – decrease

4. **(A)** If the price of a commodity increases by X%, then reduction in consumption, so as not to increase expenditure is: $\frac{100X}{100+X}\%$

   (B) If the price of a commodity decreases by X%, then increase in consumption, so as not to decrease expenditure, is: $\frac{100\,X}{100\,-X}\%$

Example

Coconut oil is now being sold at Rs. 27 per kg. During last month its cost was Rs. 24 per kg. Find by how much % a family should reduce its consumption, so as to keep the expenditure the same.

Sol. New rate = Rs. 27/kg

Original rate = Rs. 24/kg.

$\Rightarrow$ % change in rate = $\frac{27-24}{24} \times 100\% = \frac{100}{8}\%$

For fixed expenditure, % change in consumption =

$$\frac{\%\ \text{change in rate}}{100 + \%\ \text{change in rate}} \times 100$$

$$= \frac{\frac{100}{8}}{100\left[1+\frac{1}{8}\right]} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%.$$

Hence, family has to reduce its consumption by $11\frac{1}{9}\%$.

> ***Do You know ?***
>
> **Common Percentage**
>
> 100% = $\frac{100}{100}$ = 1
>
> 75% = $\frac{75}{100}$ = $\frac{3}{4}$
>
> 50% = $\frac{50}{100}$ = $\frac{1}{2}$
>
> 25% = $\frac{25}{100}$ = $\frac{1}{4}$
>
> 1% = $\frac{1}{100}$ = 0.01
>
> 0% = $\frac{0}{100}$ = 0.0

(C) Let the present population of a town be "p" and let there be an increase of X% per annum. Then

(i) Population after n years = $p[1 + (X/100)]^n$

(ii) Population n years ago = $p/[1 + (X/100)]^n$

(D) If the population of a town (or value of a machine) decreases at R% per annum, then:

(i) Population (or value of machine) after n years = $p[1-(R/100)]^n$

(ii) Population (or value of machine) n years ago = $p/[1 - (R/100)]^n$

(E) (i) If A's income is r% more than B's then B's income is $[r / (r + 100)] \times 100$ % less than A's

(ii) If A's income is r% less than B's then B's income is $[r /(100 - r)] \times 100\%$ more than A's

(F) Average percentage rate of change over a period

$= \frac{(\text{New value} - \text{Old value})}{\text{Old Value}} \times \frac{100}{n}\%$, where n = time period

(G) The percentage error = $\frac{\text{The Error}}{\text{True Value}} \times 100\%$

(H) Let us consider a product of two quantities A = a $\times$ b

If a & b change (increase or decrease) by a certain percentage say x & y respectively, then the overall %age change in their product is given by the formula: $x + y + \frac{xy}{100}$.

This formula also holds true if there are successive changes as in the case of population increase or decrease. But care has to be taken when there are either more than 2 successive changes or there is a product of more than 2 quantities as in the case of volume. **In these cases we have to apply the same formula twice.**

(i) If there is successive increase of x% and y%, then the net change will be

$$x + y + \frac{xy}{100}\%$$

(ii) If there is successive discount of x% and y%, then the total discount will be.

$$x + y - \frac{xy}{100}\%$$

(iii) (a) If there is x% increase and then x% decrease, then the net change

$$-\frac{x^2}{100}\%$$

(b) If the values are different, then net change

$$x - y - \frac{xy}{100}$$

(I) If x% of a quantity is taken by the first person, y% of the remaining quantity is taken by the second person, and z% of the remaining is taken by the third person and if A is left, then initial quantity was

$$= \frac{A \times 100 \times 100 \times 100}{(100 - x)\ (100 - y)\ (100 - z)}$$

The same concept we can use, if we add something, then the initial quantity was

$$= \frac{A \times 100 \times 100 \times 100}{(100 + x)\ (100 + y)\ (100 + z)}$$

(J) If x is A% of Z, and y is B% of z, then x is $\frac{A}{B} \times 100\%$ of y, and y is $\frac{B}{A} \times 100\%$ of x.

CONCEPT OF PROFIT & LOSS

Definition

Cost Price: CP is the price at which one buys anything

Selling Price: SP is the price at which one sells anything

Profit/Loss: This is the differential between the selling price and the cost price. If the differential is positive it is called the profit and if negative it is called as loss.

Profit/loss %: This is the profit/loss as a percentage of the CP.

Margin: Normally are in % terms only. This is the profit as a percentage of SP.

Marked Price: This is the price of the product as displayed on the label.

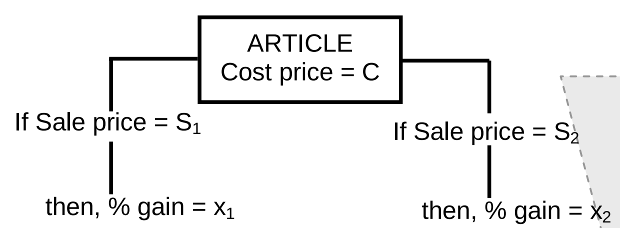
Discount: This is the reduction given on the marked price before selling it to a customer. If the trader wants to make a loss he can offer a discount on the cost price as well !

Markup: This is the increment on the cost price before being sold to a customer.

Formulae

- Gain = (S. P. – C. P.), Loss = (C. P. – S. P.)
- Gain % = (Gain × 100)/C. P, Loss % = (Loss × 100)/C. P.
- Given the cost & the gain percent, S. P. = (100 + gain %) × C. P. / 100
- Given the cost & the loss percent, S. P. = (100 – loss %) × C. P. / 100
- Given the S. P. & the gain percent, C. P. = (100 × S. P.) / (100 + gain %)
- Given the S. P. & the loss percent, C. P. = (100 × S. P.) / (100 – loss %)

An article sold at two different selling price but at same cost price



$$\frac{S_1}{100+x_1} = \frac{S_2}{100+x_2} = \frac{C}{100} = \frac{S_1 - S_2}{x_1 - x_2}$$

where x_1 or x_2 are (–) ve, if it indicates a loss, otherwise it is (+) ve.

Example

A person sells 36 oranges per rupee and suffers a loss of 4%. Find how many oranges per rupee to be sold to have a gain of 8%?

Sol. Selling price per orange = S_1 = Rs. $\frac{1}{36}$

Loss % = x_1 = – 4%

Let the selling price in second case = S_2

Gain % = x_2 = 8%

We know

$$\frac{S_1}{100 + x_1} = \frac{S_2}{100 + x_2}$$

$$\frac{\frac{1}{36}}{100 - 4} = \frac{S_2}{100 + 8} \qquad \Rightarrow S_2 = \frac{1}{36} \times \frac{1}{96} \times 108 = \frac{1}{32}$$

$\therefore$ He sells 32 oranges per rupee.

Alternate method:

In these cases always, find the unit price, i.e. for one orange. Here, Sale price per orange = Rs $\frac{1}{36}$ = S_1

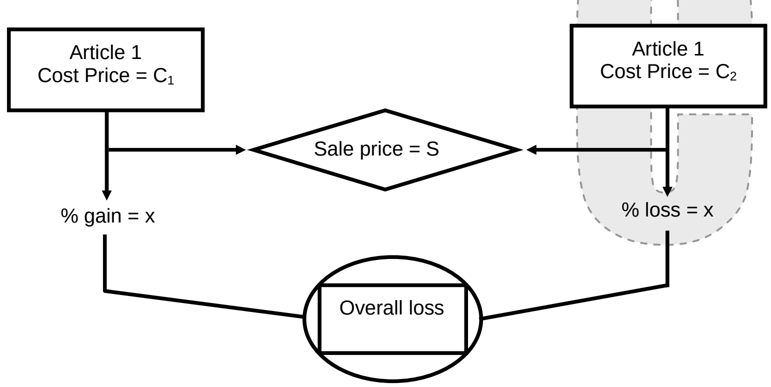
Now if the cost price per orange is X, then 0.96 X = 1/36,

Thus X = 1/(36 × 0.96)

And selling price per orange to gain 8% = 1.08 × 1/(36 × 0.96) = 1/32

$\therefore$ He sells 32 oranges per rupee.

Two different articles sold at same selling price



Overall % loss = $-\left(\frac{x}{10}\right)^2$ %

Example

Each of the two horses is sold for Rs. 720. The first one is sold at 25% profit and the other one at 25% loss. What is the % loss or gain in this deal?

Sol. Total selling price of two horses = 2 × 720 = Rs. 1,440

The CP of first horse = $\frac{S.P. \times 100}{(100 + P)} = \frac{720 \times 100}{(100 + 25)}$ = Rs. 576

The CP of second horse = $\frac{S.P. \times 100}{(100 - L)} = \frac{720 \times 100}{(100 - 25)}$ = Rs. 960

Total CP of two horses = 576 + 960 = Rs. 1,536

Therefore, loss = Rs. 1,536 – Rs. 1,440 = Rs. 96

$\therefore$ % loss = $\frac{96 \times 100}{1536}$ = 6.25%.

Shortcut : % loss = $\left(\frac{25}{10}\right)^2$ = 6.25% [From formula]

CONCEPT OF SIMPLE & COMPOUND INTEREST

Simple Interest

Definition

If a person A borrows some money from another person B for a certain period, then after that specified period, the borrower has to return the money borrowed as well as some additional money. This additional money that borrower has to pay is called ***interest***. The actually borrowed money by A is called principal (SUM). The principal and the interest together is called ***amount***. The interest that the borrower has to pay for every 100 rupees borrowed for every year is known as rate per cent per annum. It is denoted as R% per annum = $\frac{R}{100}$.

The time for which the borrowed money has been used is called the ***time***. It is denoted as T years. The interest is directly proportional to the principal, the rate and time for which the borrowed sum is used.

If the interest on a certain sum borrowed for a certain period is reckoned uniformly, then it is called Simple Interest and denoted as S.I.

$$\therefore \text{ Simple Interest (S.I.)} = \frac{PxRxT}{100}$$

Where P = Principal or the sum borrowed

R = Rate per cent per annum

T = Number of years for which the borrowed money has been used.

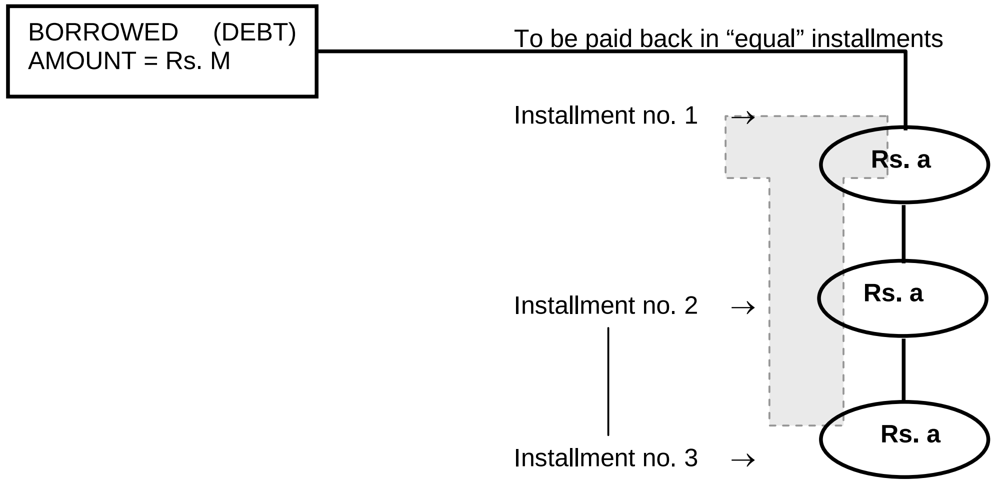
Amount

PRINCIPAL (SUM) $\rightarrow$ Actually borrowed money	+	**SIMPLE INTEREST** $\rightarrow$ Interest accrued on the sum borrowed	=	**AMOUNT** $\rightarrow$ money to be returned by the borrower.
P		SI		A

$$\therefore A = P + SI = P + \frac{PRT}{100}$$

$$A = P\left[1 + \frac{RT}{100}\right]$$

REPAYMENT OF DEBT IN EQUAL INSTALLMENTS



$$\text{BORROWED AMOUNT (DEBT)} = M = na + \frac{ra}{100 \times Y} \times \frac{n(n-1)}{2}$$

where, r = rate of interest per annum

Y = no. of installments per annum

Y = 1, when each installment is paid yearly

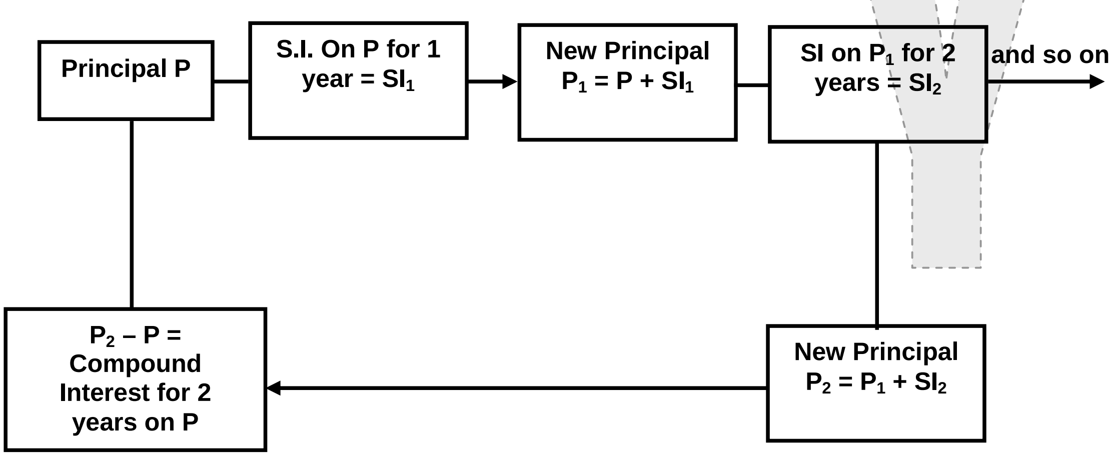
Y = 2, when each installment is paid half-yearly

Y = 4, when each installment is paid quarterly

Y = 12, when each installment is paid monthly

Compound Interest

☞ As discussed in the topic on 'Simple Interest', the principal (P) remains constant throughout the period for which the money (principal) is borrowed. But, when the borrower fails to pay the principal as it falls due, the interest for the first year (conversion period) is added to the original principal at the end of the first year (conversion period for charging the interest) and this sum (P + 1^{st} year interest on P) becomes the principal for the second year and so on.



Hence for every changing year, the principal goes on changing and accordingly the amount of interest accrued on varying principal will be different in every year. The money lent under this condition is charged with Compound Interest.
Since, principal increases after every year (reckoning period), the amount of interest in Compound Interest is always more than Simple Interest.

While solving the problems on Compound Interest, it is assumed that Interest is compounded yearly, unless otherwise specified.

Toolkit

(a) $A = P\left[1+\frac{R}{100\times n}\right]^{n\times t}$

where, R = rate per cent year (% p.a.)
= rate per Rs. 100 per year,
t = time in years,
n = number of conversions per year,
A = Amount
But, in general the interest is compounded yearly, so n = 1

$\therefore \quad A = P\left(1+\frac{R}{100}\right)^{t}$

(b) Compound Interest (CI) = A – P

$= P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t}-1\right]$

but, when the interest is compounded yearly, n = 1

$C.I = P\left[\left(1+\frac{R}{100}\right)^{t}-1\right]$

Difference between CI & SI

If R = Rate per cent per annum = Rupee per hundred per annum,
T = time in year,
P = Principal, then,

$S.I. = \frac{P\times R\times T}{100}$

and $C.I. = P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t}-1\right]$

Now, C.I. is greater than S.I when $n \times T > 1$

$\therefore \quad C.I - S.I = P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t}-\frac{RT}{100}-1\right]$

Case 1: When T = 2 years and n = 1 per year.

$$(C.I. - S.I.) = P\left(\frac{R}{100}\right)^2$$ (a relation with P and R)

and $$(C.I. - S.I.) = \frac{R \times S.I.}{2 \times 100}$$ (a relation with SI and R)

Case 2: When T = 3 years and n = 1 per year

$$(C.I. - S.I.) = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$$ (a relation with P and R)

and $$(C.I. - S.I) = \frac{S.I}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$$ (a relation with SI and R)

The following points need to be observed in relation to SI & CI:

- SI remains constant every year as the principal remains the same every year
- Principal for CI increases constantly as the interest is added on to the principal at the end of the year & the same becomes the principal for the next year.

Equal annual installment

Let the value of each equal annual installment = Rs. a

Rate of interest = R % p.a.

Number of installments per year = n

Number of years = T

$\therefore$ Total number of installments = $n \times T$

Borrowed Amount = B

then,

$$a\left[\frac{100}{100+R} + \left(\frac{100}{100+R}\right)^2 + \ldots\ldots\ldots + \left(\frac{100}{100+R}\right)^{n \times T}\right] = B$$